A more rigorous description of the multiplierless MPPT principle.

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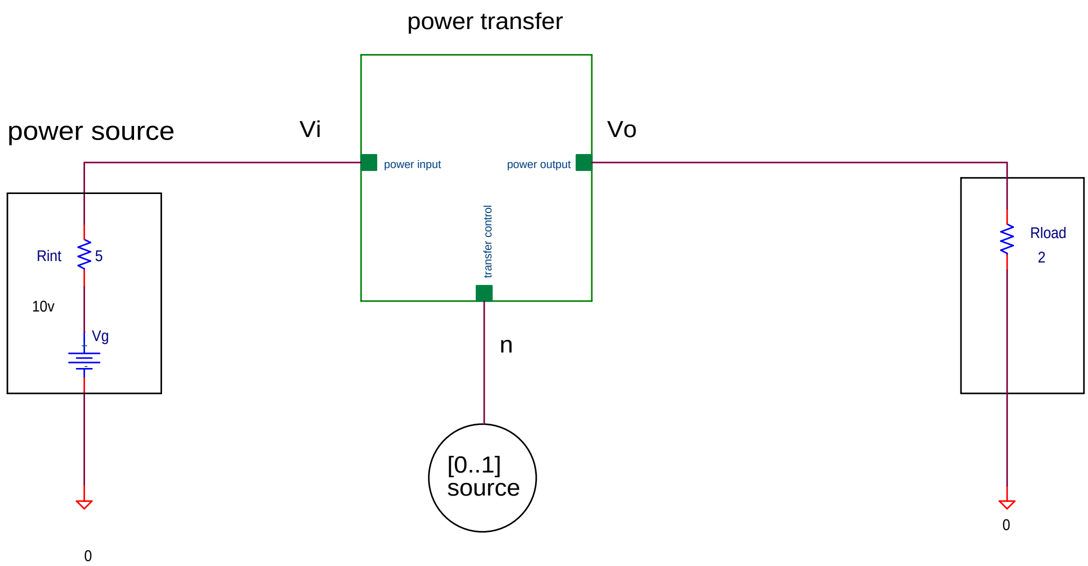
Let’s assume the simple circuit described in the article (idea for design #2394) as our environment:



There is some minor difference: there is no the MPPT block, and the load is only a resistor.

As written in the article, the power transfer block (regardless to its practical implementation) is a kind of ‘dc transformer’ where

$$W_{input} = W_{output} \qquad (1)$$

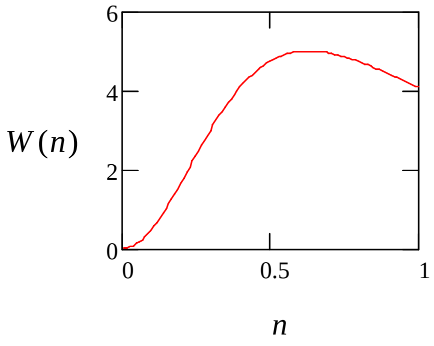
$$V_o = V_i \cdot n \qquad \text{where} \qquad 0 \le n \le 1 \qquad (2)$$

so the power transfer acts like a transformer, where

$$I_o = I_i \cdot \frac{1}{n} \qquad (3)$$

The power supplied to the power transfer may be written as a function of n.
From (2) and (3) the impedance at the power transfer input is: $Z_i = \frac{R_{load}}{n^2}$ the power dissipated by Z_i is

$$W = \frac{\left(\frac{\frac{R_{load}}{n^2}}{\frac{R_{load}}{n^2} + R_{\text{int}}} \cdot V_g \right)^2}{\frac{R_{load}}{n^2}}$$



The plot on the right is obtained using the variable values shown in the block diagram.
There is an n that maximizes the power. Its value can be found by equating the derivative of the power to 0:

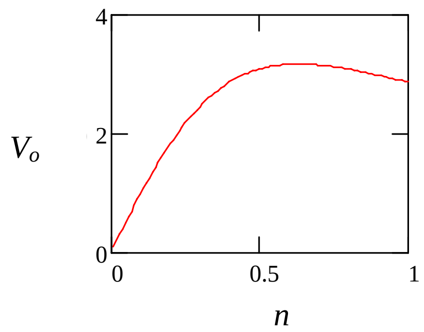
$$\frac{dW}{dn} = V_g^{\,2} \cdot (-2) R_{load} \cdot n \cdot \frac{n^2 \cdot R_{\text{int}} - R_{load}}{\left(R_{load} + n^2 \cdot R_{\text{int}}\right)}$$

Solving the equation (and skipping trivial and negative results) we have the best value of n:

$$n = \sqrt{\frac{R_{load}}{R_{\text{int}}}}$$

This was calculated considering the generator side. Let's take a look on the load side, measuring the voltage:

$$V_o = V_g \frac{\frac{R_{load}}{n^2}}{\frac{R_{load}}{n^2} + R_{\text{int}}} \cdot n$$



As before, the plot on the right is obtained using the variable values shown on the block diagram.

If we follow the same process, i.e. we equate the output voltage derivative to 0, we obtain

$$\frac{dV_0}{dn} = -R_{load} \cdot V_g \cdot \frac{n^2 R_{\text{int}} - R_{load}}{(n^2 R_{\text{int}} + R_{load})^2} = 0$$

the positive result of this equation is still $n = \sqrt{\dfrac{R_{load}}{R_{\text{int}}}}$

which demonstrates the possibility of using the voltage output as MPPT control parameter.

MPPT with a synchronous modulator.

The MPPT goal is to find the maximum of the power output (W) vs. the control signal (n), $W(n)$. An conventional feedback cannot be used because $W(n)$ is not a generally growing curve. But its derivative can be used. $dW(n)/dn$ crosses 0 when $W(n)$ has a maximum (fig. 1).

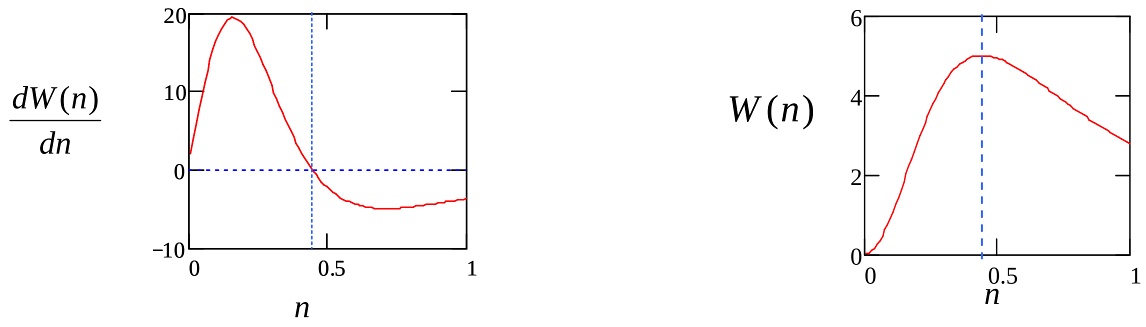


Fig. 1 Although it contains the information about the maximum power position, the power curve w(n) (right) cannot be directly used to drive a feedback loop to seek the maximum. Its derivative (left) can. Despite its funny shape, it is linear around the 0 crossing (when the power has a maximum), and has a single 0-crossing point in the interval of existence of n.

A synchronous modulation demodulation process is represented in fig.2

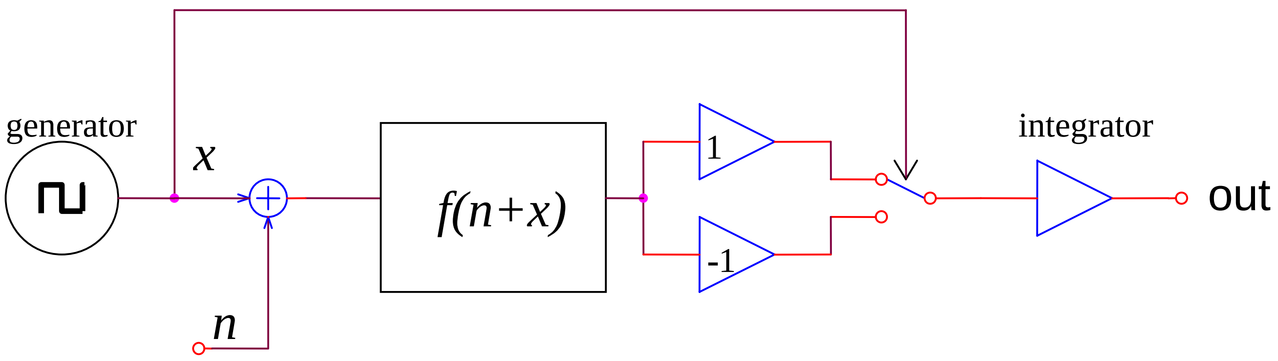


Fig.2 Synchronous modulation-demodulation scheme. The transfer block ***f*** is fed with a modulation signal, ***x***, and an external signal, ***n*** (***x*** frequency>>***n*** frequency) . The controlled switch feeds the integrator with ***f(n+x)*** when ***x>0*** and with ***–f(n+x)*** when ***x<0***.

The schematic in fig.2 may be expressed by the equation: [1]

$$out(n,\Delta x) = \int_{-\Delta x}^{\Delta x} f(n+x)\frac{x}{|x|}dx$$

We may demonstrate that: [2]

$$\lim_{\Delta x \to 0} \frac{1}{(\Delta x)^2} \int_{-\Delta x}^{\Delta x} f(n+x)\frac{x}{|x|}dx = \left.\frac{df(y)}{dy}\right|_{y=n} \qquad (4)$$

This shows that the circuit in fig.2 computes the derivative of the curve ***f(y), y=n*** .
If we use this strategy on the curve ***W(n)*** we may close the MPPT loop on the power derivative, so obtaining a stable maximum power finder. Note that the schematic in fig.2 is identical to the schematic shown in the article (fig. 3), where the multiplier inside the MPPT does the same job of the switch (assuming a symmetrical square wave generator).

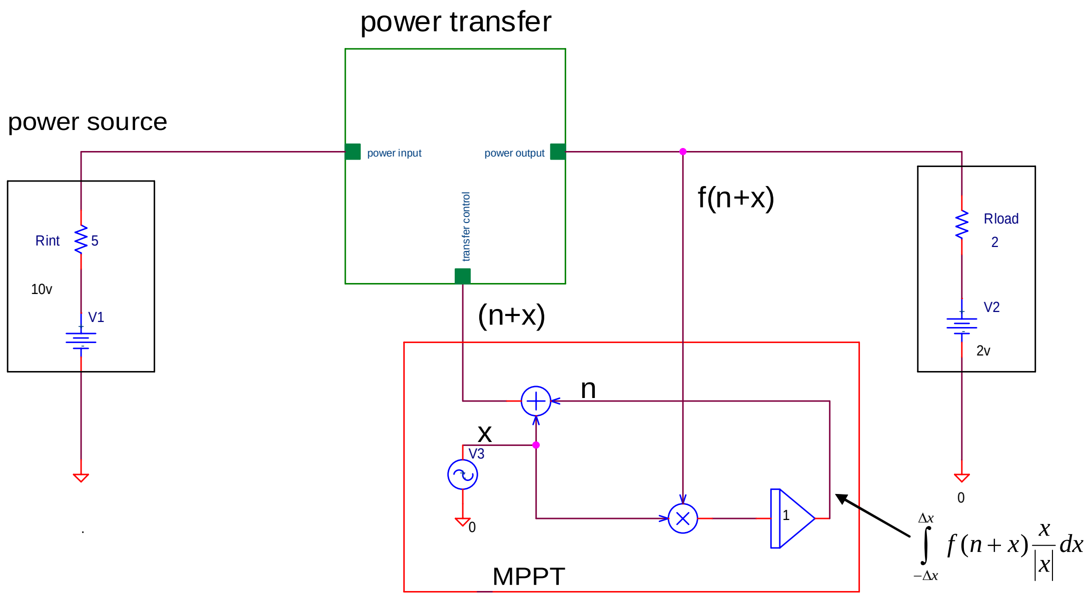


Fig 3. The schematic in the idea for design #2394, identifying some signal paths. The feedback loop is closed by feeding ***n*** with $\int_{-\Delta x}^{\Delta x} f(n+x)\frac{x}{|x|}dx$

[1] From an electronic designer point of view this may appear unusual, since there is no time reference. That's true. The equation exists without the time (we want to compute the ***f(n)*** derivative despite the physical nature of the variable) but, in a real circuit, time is usually the integration variable. Defining x as a time function (i.e. *x(t)=at*) the integration operation becomes a time integration. Obviously, to implement a real circuit x(t) must be periodic an the integration operation has to be extent over the period.

[2] Demonstration:

$$\int_{-\Delta x}^{\Delta x} f(n+x)\frac{x}{|x|}dx = \int_{0}^{\Delta x} f(n+x)dx - \int_{-\Delta x}^{0} f(n+x)dx$$

assuming: $\frac{d(g(x))}{dx} = f(x)$

$u = n + x$

$$\int_{n}^{n+\Delta x} f(u)du - \int_{n-\Delta x}^{n} f(u)du = g(u)\Big|_{n}^{n+\Delta x} - g(u)\Big|_{n-\Delta x}^{n} = g(n+\Delta x) + g(n-\Delta x) - 2g(n)$$

from (4), and using De L'Hôpital's rule

$$\lim_{\Delta x \to 0} \frac{1}{(\Delta x)^2}\left(g(n+\Delta x)+g(n-\Delta x)-2g(n)\right) = \lim_{\Delta x \to 0} \frac{\frac{d}{d\Delta x}\left(g(n+\Delta x)+g(n-\Delta x)-2g(n)\right)}{\frac{d}{d\Delta x}(\Delta x)^2} = \lim_{\Delta x \to 0} \frac{f(n+\Delta x)-f(n-\Delta x)}{2\Delta x} = \frac{df(n)}{dn}$$